Abstracts

Modular forms, p-adic L-functions and Selmer groups
July 7-13, 2013 - NIO (Oriahovitza), Bulgaria

Denis Benois : *On extra-zeros of p-adic L-functions*

Abstract: In this talk we discuss the algebraic machinery which allows to formulate some general conjectures about extra zeros of p-adic L-functions.

John Bergdall : *Parabolizations over families of trianguline representations*

Abstract: Since their genisis, p-adic families of automorphic forms have played a key role in answering (and asking) a multitude of deep arithmetic questions. One indispensible technique is to understand the natural family of Galois representations appearing thereupon, with the structure at p playing a distinguished role. Following ideas of Kisin and Colmez we now know that the relevant representations are all trianguline, a term referring to the associated (φ, Γ)-module. In this talk we present new results on this variation: the non-critical steps (to be defined) of the triangulation at a classical point can be realized in an affinoid neighborhood of that point. As a corollary, we deduce triangulations on affinoid neighborhoods of an open Zariski dense locus of classical points. We also can give new results regarding the smoothness of the family at *certain* so-called critical points.

Tobias Berger : *An analogue of Shintani's theta lift for imaginary quadratic fields*

Abstract: I will first describe how to adapt the work of Johnson-Leung and Roberts to imaginary quadratic fields, i.e. how to lift Bianchi modular forms to Siegel paramodular forms of genus 2. Then I will discuss work in progress on relating the Fourier coefficients of this lift to L-values of the Bianchi modular form.

Gebhard Boeckle : *Images of p-adic Galois representations*

Abstract: For an elliptic curve E without complex multiplication over a number field K, in 1972 Serre proved (1) that the image of the absolute Galois group of K when acting on the ℓ-adic Tate-module of E is an open subgroup of $GL_2(\mathbb{Z}_\ell)$, and (2) that the images for different ell are 'almost independent" by proving an adelic openess theorem. Already for abelian varieties over $\mathbb{Q}$ the generalization of (1) is rather difficult and still open.

Recently, a variant of (2), the question of almost independence has been settled in great generality, again starting with work of Serre. Here one considers for a scheme X of finite type over a field K the family of ℓ-adic cohomology groups $V_\ell = H^q_c(X, \mathbb{Q}_\ell)$ as finite dimensional representations of the absolute Galois group G_K of K when ℓ varies over the rational primes. Various independence results in this setting are due to Serre, Gajda-Petersen, and Gajda-Petersen and myself.

In the talk I shall give a general introduction to the theme of images of Galois representations of certain geometric objects and, in the second half, focus on the results with Gajda and Petersen.

Ehud de Shalit : *L-invariants of p-adically uniformized varieties*

Abstract: p-adically uniformized varieties are varieties that admit a rigid analytic uniformization by the Drinfel'd p-adic symmetric domain. Among such varieties lie interesting examples of Shimura varieties attached to unitary groups. Following recent advances in understanding the cohomology of the Drinfel'd space and its quotients we shall explain how to attach to such a d-dimensional variety a sequence of L-invariants. We shall also explain how Besser's theory of p-adic integration yields a transcendental construction of these invariants. We will speculate on possible relations between these L-invariants and special values of p-adic L-functions attached to modular forms on unitary Shimura varieties. This talk is about work in progress with Amnon Besser.

Adam Gamzon : *Unobstructed Hilbert modular deformation problems*

Abstract: Let $\rho_{f,\lambda}$ be the Galois representation associated to a Hilbert newform f of parallel weight $k > 2$ and consider its semisimple reduction $\bar{\rho}_{f,\lambda}$. We will discuss how the universal ring for deformations of $\bar{\rho}_{f,\lambda}$ with fixed determinant is unobstructed for almost all primes λ. The problem comes down to showing that various local invariants vanish at all places dividing the characteristic or the level of the newform. The first half of the talk will be devoted to a brief review of Galois deformation theory and a review of analogous results for classical modular forms. In the second half of the talk, we will outline a proof, giving some particular attention to the calculation of the local invariants at places dividing the characteristic.

Robert Harron : *Computing Hida families using overconvergent modular symbols*

Abstract: Given that the ability to experiment with modular forms on a computer has been a very useful tool (and so has Hida theory!), we are developing a way to compute with Hida families. Glenn Stevens' invention of overconvergent modular symbols was an important step in studying p-adic aspects of modular forms numerically (such as in the work of Pollack–Stevens). Our approach is to study p-adic families of overconvergent modular symbols. This allows us, among other things, to compute q-expansions of Hida families, the structure of the ordinary p-adic Hecke algebra, two-variable p-adic L-functions, and L-invariants. The first part of the talk will be introductory, discussing, for instance, how one represents modular forms on a computer. Then, I'll move on to describe our new work and give several examples of what we can do.

This is ongoing joint work with Rob Pollack, Evan Dummit, Marton Hablicsek, Lalit Jain, and Daniel Ross.

Haruzo Hida : *Arithmetic of Weil numbers and Hecke fields*

Abstract: Analyzing prime factorization of Weil numbers in the union of algebraic extensions with bounded degree of the cyclotomic field K of all p-power roots of unity, we show that there are only finitely many Weil p-numbers of a given weight for a prime p (upto roots of unity). Applying this fact to Hecke eigenvalues of cusp forms in a p-adic analytic families of cusp forms of p-power level, we show that the field generated by the eigenvalues over the family has unbounded degree over K.

Fabian Januszewski : *On the non-vanishing of periods for $GL(n)\times GL(n-1)$*

Abstract: I will discuss an approach to proving the non-vanishing of the periods occuring in the study of special values of Rankin-Selberg L-functions of $GL(n) \times GL(n-1)$ as in the work of Kazhdan-Mazur-Schmidt, Kasten-Schmidt and myself. The method relies on cohomological induction and an appropriate multiplicity-one theorem.

Dimitar Jetchev : *Euler Systems from CM cycles for Unitary Shimura Varieties and the Gross-Prasad Conjectures*

Abstract: Euler systems have been invented by Kolyvagin as a tool to algebraically model L-functions and have been successfully used in proving various deep results towards the Birch and Swinnerton-Dyer conjecture, the Iwasawa main conjecture and various modularity theorems. Currently, there are only few constructions of Euler systems known in the literature: the Euler systems of cyclotomic units, Stickelberger elements, elliptic units, Siegel units (Kato's construction) and Heegner points (Kolyvagin's construction). It is an open question to understand more conceptually the construction of Euler systems and to place it in a more general-representation theoretic context. In this talk, we discuss a novel, higher-dimensional construction of an Euler system from CM 1-cycles on certain Shimura varieties for the group $U(2,1)\times U(1,1)$ via the Gross-Prasad restriction problem for the Gelfand pair $U(1,1)$ embedded diagonally in $U(2,1)\times U(1,1)$. This construction can be used to prove new results towards a generalization of the Birch and Swinnerton-Dyer conjecture closely related to a recent Gross-Zagier type formula studied by Zhang-Zhang-Yuan in the same case. In addition, it indicates a general strategy for constructing Euler systems out of restriction problems for automorphic representations in the context of general recent conjectures by Gan-Gross-Prasad.

Andrei Jorza : *L-invariants for symmetric powers of Hilbert modular forms*

Abstract: To a Hilbert modular form one may attach a p-adic analytic L-function interpolating certain special values of the usual L-function. Conjectures in the style of Mazur, Tate and Teitelbaum prescribe the order of vanishing and first Taylor coefficient of such p-adic L-functions, the first coefficient being controlled by an L-invariant which has conjectural (arithmetic) value defined by Greenberg and Benois. I will explain how to compute arithmetic L-invariants for symmetric powers of Iwahori level Hilbert modular forms using Langlands functoriality and triangulations of (phi, Gamma)-modules on eigenvarieties. This is based on joint work with Robert Harron.

Matteo Longo : *A refined Beilinson-Bloch Conjecture for motives of modular forms*

Abstract: I will discuss a refined version of the Beilinson-Bloch conjecture for the motive associated with a modular form of even weight. This conjecture relates the dimension of the image of the relevant p-adic Abel-Jacobi map to certain combinations of Heegner cycles on Kuga-Sato varieties. I will expose some results in the direction of the conjecture and, in doing so, obtain higher weight analogues of results for elliptic curves due to H. Darmon. Joint work with S. Vigni.

Giovanni Rosso : *Trivial zero for the symmetric square of an elliptic curve over a totally real field*

Abstract: Let p be a prime, in the 90's Greenberg formulated a very nice conjecture concerning the values of the derivative of the p-adic L-function of a motive M when the so-called trivial zeros appear. The aim of the talk is to present in detail this conjecture and to give a proof of it in the particular case of the symmetric square of a modular elliptic curve E over a totally field F, when p is inert in F and E has multiplicative reduction at p (plus some other small hypothesis on the conductor). This work is a generalization of (unpublished) work of Greenberg and Tilouine.

Victor Rotger : *Stark-Heegner points and iterated p-adic integrals*

Abstract: One of the most intriguing questions on the arithmetic of elliptic curves is the construction of non-torsion rational points over a given number field. In this talk I will describe joint work in progress with H. Darmon and A. Lauder, where we propose a new conjectural formula expressing the logarithms of certain Stark-Heegner points to an iterated p-adic integral associated to the choice of two weight one p-stabilized ordinary eigenforms g and h. The points in question ought to be rational over the field cut out by the tensor product of the two Artin representations attached to g and h. This formula can be proved in some cases when both g and h are theta series associated to the same imaginary quadratic field and the points are classical Heegner points. But our conjecture also encompasses less classical settings where the points are rational over more interesting fields, like cyclotomic fields, abelian extensions of real quadratic fields or A_4, S_4, A_5-extensions of $\mathbb{Q}$. We hope to test numerically our formula in some cases.

Eric Urban and Christopher Skinner Mini-course : Vanishing of L-functions and rank of Selmer groups for rational elliptic curves

Abstract: The goal of this mini course is to sketch the main arguments used in the proof that the Selmer group of a rational elliptic curve is of (co)-rank at least one whenever its L-function vanishes at $s = 1$. In the first lecture, we will explain the Galois deformation argument. In the second one, we will introduce certain Eisenstein series on $U(2,2)$ and their p-adic families. In the last lecture, we will introduce families of nearly holomorphic forms and use Eigenvarieties to construct the automorphic family that is used to construct the Galois deformation of the first lecture.

Christopher Skinner : *P-adic numbers and their uses in number theory*

Abstract: This talk is meant for a general mathematical audience. It will introduce the p-adic numbers and describe some of the ways they have been used to resolve a variety of problems in number theory, including - as time permits - describing the zero set of a recursion sequence, understanding rational points on curves, proving modularity of elliptic curves, and studying special values of L-functions.

Jacques Tilouine : *Big image of automorphic Galois representations and congruence ideals*

Abstract: In a work in progress with H. Hida, we show the existence of a principal congruence subgroup contained in the image of the Galois representation associated to a p-adic family for $GSp(4)$. This defines an ideal, called Galois level ; we relate this ideal with congruence ideals.

Jeanine Van Order : *Galois averages of automorphic L-functions*

Abstract: I will present some results about the generic nonvanishing of central values of certain Rankin-Selberg L-functions for GL(2), where the notion of Galois conjugacy plays a central role. In particular, I will explain what can be deduced from (i) the algebraicity theorems of Shimura, (ii) the existence of related p-adic L-functions, and (iii) certain auxiliary averaging arguments. If time permits, then I will also describe some problems for the general setting of automorphic L-functions.

Sarah Zerbes: *Euler systems for Rankin-Selberg convolutions*

Abstract: An Euler system is a certain compatible family of classes in the cohomology of a Galois representation, which plays a key role in relating arithmetical properties of the representation to values of the associated L-function. Only a few examples of such systems have been constructed to date, although they are conjectured to exist in quite general settings. I will describe a construction of an Euler system for the tensor product of the Galois representations of two modular forms, and an application to bounding Selmer groups. This is joint work with Antonio Lei and David Loeffler.